

Efficient Bilateral Trade

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December 9, 2015

Quotation: Jackson and Sonnenschein

"Over the past fifty years we have learned that social welfare possibilities depend not only on resources and technology, but also on incentive constraints (including participation constraints) and the ability of social institutions to mediate those constraints."

Econometrica, 2007

Is Efficient Bilateral Trade Possible?

Buyer and seller privately know their values for an indivisible good

Either of the two agents may have the higher value

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This negative answer presumes quasilinear utilities.

Is Efficient Bilateral Trade Possible?

Quasi-linearity is quite restrictive

- means only gains from trade are those from assigning object to person who "values" it most
- efficiency means "right" person gets the good
- presumes consumption value of item does not depend on other things: e.g. money holdings

Without quasi-linearity there can be efficiency gains associated with the transfer of money

Our Contribution

Efficient trade is possible if

- The good is normal (each agent's reservation price for the good increases with the agent's money holding, Cook and Graham, 1977).

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Efficient trade is possible if

- The good is normal (each agent's reservation price for the good increases with the agent's money holding, Cook and Graham, 1977).
- Agents' utilities are not too responsive to their private information (or, else, the asymmetry of information is not too large).
- The elasticities of the marginal utilities of money and good with respect to private information are well-behaved.

Literature on Efficient Trade

- Disjoint domains of types (Myerson and Satterwaite 1983)
- Infinite risk-aversion (Chatterjee and Samuelson 1983)
- Correlated types (McAfee and Reny 1992)
- Ownership not too asymmetric (Cramton, Gibbons, and Klemperer 1987)

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- Many goods (Jackson and Sonnenschein 2007, Jackson, Sonnenschein, and Xing 2014)

Random Mechanisms

Garratt (1999)

- shows that random mechanisms can dominate deterministic ones in a complete information setting

Baisa (2013)

- shows expected revenues from a random mechanism exceed the expected revenues from standard auction formats when number of bidders is sufficiently large
- provides an example of a profile of utility functions such that no strategy-proof, individually rational, non-subsidized mechanism allocates the good in an efficient way.

Following example shows that in some settings efficient trade can be accomplished in strategy-proof way; not true generally

Example: Shifted Cobb-Douglas

Utility $U(x, m; \theta) = (1 + \theta x) m$ where

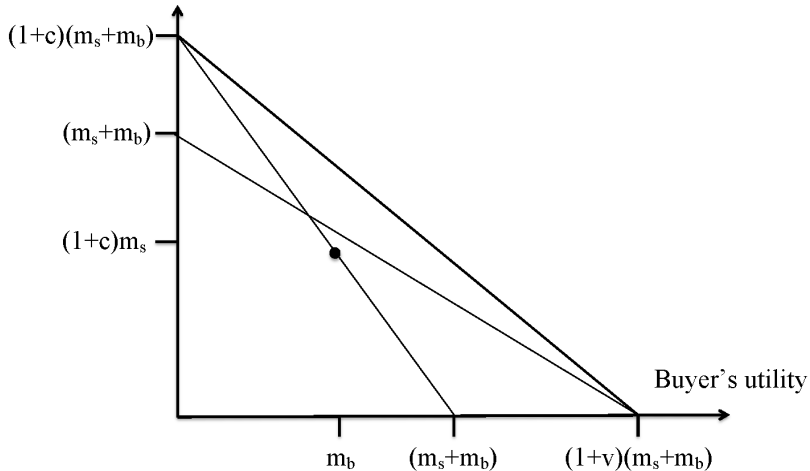
$x = 1$ if the agent has the good, or $x = 0$ otherwise;

$m \geq 0$ money holdings of the agent; m_s, m_b initial money holdings;

$\theta \geq 0$, agent's privately known type, distributed arbitrarily (correlation allowed but not needed).

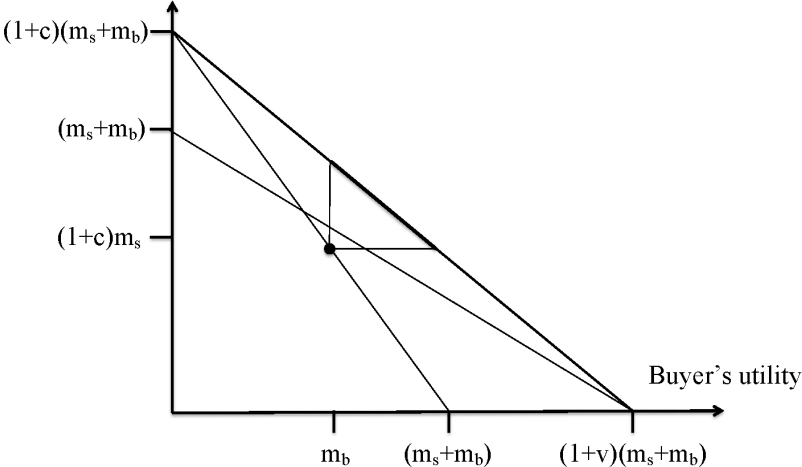
Example: Pareto Frontier

Seller's utility



Example: IR Set on the Pareto Frontier

Seller's utility



Example: Efficient Mechanism

Give the good and all money to the seller with probability $\frac{m_s}{m_s+m_b}$,

Give the good and all money to the buyer with probability $\frac{m_b}{m_s+m_b}$.

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Example: Efficient Mechanism

Give the good and all money to the seller with probability $\frac{m_s}{m_s + m_b}$,

Give the good and all money to the buyer with probability $\frac{m_b}{m_s + m_b}$.

- Incentive compatible and efficient
- Individually rational for the seller:

$$\frac{m_s}{m_s + m_b} (1 + c) (m_s + m_b) \geq (1 + c) m_s$$

- Individually rational for the buyer:

$$\frac{m_b}{m_s + m_b} (1 + v) (m_s + m_b) \geq m_b$$

Model

Total amount of money M fixed throughout.

Endowments:

- seller's: the indivisible good and money m_s .
- buyer's: money $m_b = M - m_s$.

Utility $u(x, m; \theta)$

- strictly increasing in x , m , and θ ,
- strictly concave in m , and
- twice differentiable in m and θ .

Privately known types c, v ; arbitrary continuous distribution.

Normality: Cook and Graham

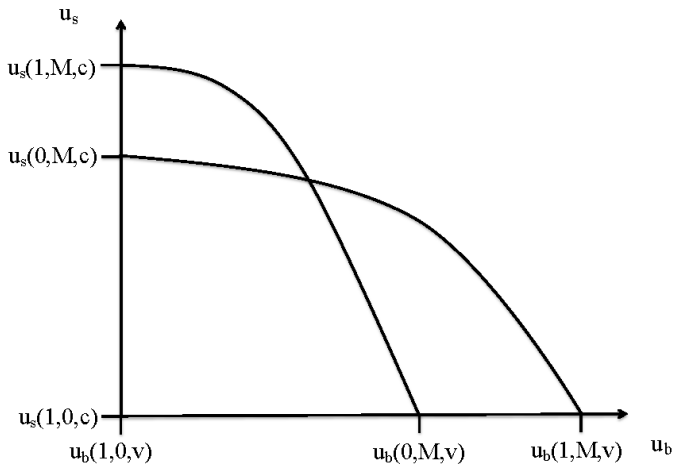
The indivisible good is normal for θ if for any $m, p, \epsilon > 0$:

$$u(0, m; \theta) = u(1, m - p; \theta) \implies u(0, m + \epsilon; \theta) < u(1, m + \epsilon - p; \theta).$$

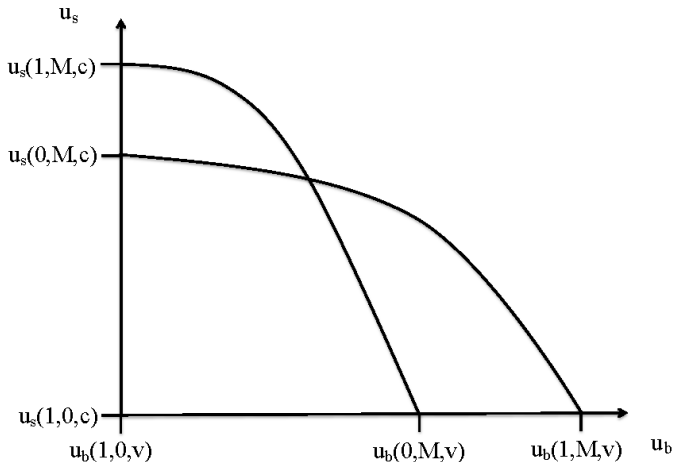
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Normality: Cook and Graham



An example to keep in mind: $u(x, m; \theta) = \theta x + V(m)$

A Condition on How Private Information Affects Utilities

$$\frac{\partial}{\partial \theta} \log (u(1, m, \theta) - u(0, m, \theta)) > \frac{\partial}{\partial \theta} \log \left(\frac{\partial}{\partial m} u(x, m, \theta) \right) = \text{constant}$$

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Analogous to single crossing property in that it guarantees F. O. approach is sufficient.

Always satisfied in the separable case: $\theta x + V(m)$.

Main Result

Fix c^* , v^* and $u(\cdot, \cdot; \cdot)$. For any initial money endowments but one, there is $\delta > 0$ such that if

$$\max_{x \in \{0,1\}, m \in [0, M], \theta} |u(x, m, \theta) - u(x, m, \theta^*)| < \delta,$$

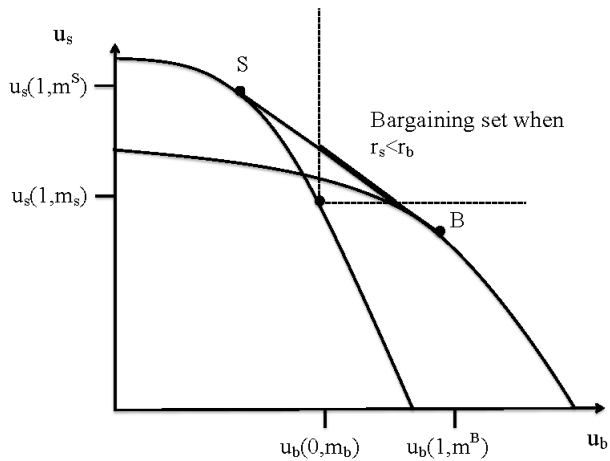
then there is an incentive-compatible and individually-rational mechanism that generates efficient trade.

Alternative Formulation

Given θ^ and any initial money endowments but one, there is a non-degenerate interval $[\underline{\theta}, \bar{\theta}] \ni \theta^*$ such that:*

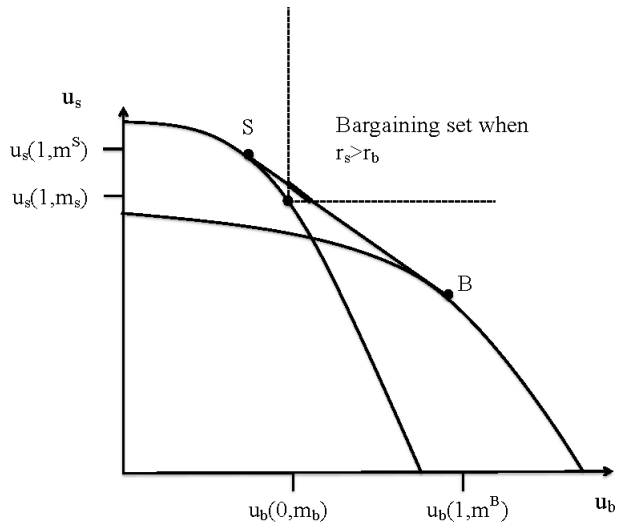
- for any distribution of agents' types on $[\underline{\theta}, \bar{\theta}] \times [\underline{\theta}, \bar{\theta}]$, there is an incentive-compatible, individually-rational mechanism that generates efficient trade.*

Commonly known types



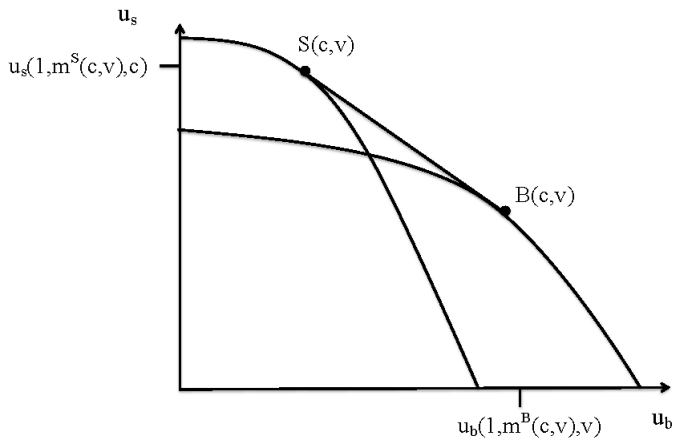
Garratt (GEB, 1999)

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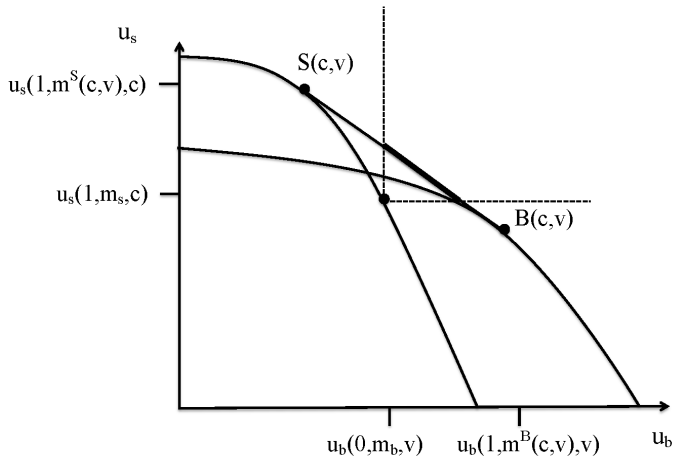


Garratt (GEB, 1999)

Pareto Frontier with private info



The Need to Elicit Types



Proof: How to Elicit Types?

Mechanism: agents obtain allocation $S(c, v)$ with probability $\pi(c, v)$ and allocation $B(c, v)$ with probability $1 - \pi(c, v)$.

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Challenge: find function $\pi(c, v)$ such that agents report their true types in Bayesian Nash equilibrium.

Step 1: we solve the agents' first order conditions to find π

Step 2: we verify the agents' second order conditions.

First Order Conditions

$$\begin{aligned}\Pi^S(c, \hat{c}) &= E_v[\pi(\hat{c}, v)u(1, m^S(\hat{c}, v), c) \\ &\quad + (1 - \pi(\hat{c}, v))u(0, M - m^B(\hat{c}, v), c)]\end{aligned}$$

is maximized at $\hat{c} = c$, and similarly for the buyer,

$$\begin{aligned}\Pi^B(v, \hat{v}) &= E_c[\pi(c, \hat{v})u(0, M - m^S(c, \hat{v}), v) \\ &\quad + (1 - \pi(c, \hat{v}))u(1, m^B(c, \hat{v}), v)]\end{aligned}$$

is maximized at $\hat{v} = v$.



First Order Conditions

$$S_1(c, v) = u(1, m^S(c, v), c) - u(0, M - m^B(c, v), c)$$

$$B_1(c, v) = u(1, m^B(c, v), v) - u(0, M - m^S(c, v), v)$$

$$S_2(c, v) = \left[\frac{\partial}{\partial m} u(1, m^S(c, v), c) \right] \left[\frac{\partial}{\partial c} m^S(c, v) \right] \\ + \left[\frac{\partial}{\partial m} u(0, M - m^B(c, v), c) \right] \left[\frac{\partial}{\partial c} m^B(c, v) \right]$$

$$B_2(c, v) = \left[\frac{\partial}{\partial m} u(1, m^B(c, v), v) \right] \left[\frac{\partial}{\partial v} m^B(c, v) \right] \\ + \left[\frac{\partial}{\partial m} u(0, M - m^S(c, v), v) \right] \left[\frac{\partial}{\partial v} m^S(c, v) \right]$$

$$\phi(c) = E_v \left\{ \left[\frac{\partial}{\partial m} u(0, M - m^B(c, v), c) \right] \left[\frac{\partial}{\partial c} m^B(c, v) \right] \right\}$$

$$\psi(v) = E_c \left\{ \left[\frac{\partial}{\partial m} u(1, m^S(c, v), v) \right] \left[\frac{\partial}{\partial v} m^S(c, v) \right] \right\}$$

First Order Conditions

$$E_v \left[S_1(c, v) \frac{\partial}{\partial c} \pi(c, v) + S_2(c, v) \pi(c, v) \right] = \phi(c)$$

$$E_c \left[B_1(c, v) \frac{\partial}{\partial v} \pi(c, v) + B_2(c, v) \pi(c, v) \right] = \psi(v)$$

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$$E_c \left[B_1(c, v) \frac{\partial}{\partial v} \pi(c, v) + B_2(c, v) \pi(c, v) \right] = \psi(v)$$

Solution

$$\pi(c, v) = b(v) \pi^B(c, v) + s(c) \pi^S(c, v)$$

where π^B and π^S solve

$$S_1(c, v) \frac{\partial}{\partial c} \pi^B(c, v) + S_2(c, v) \pi^B(c, v) = 0$$
$$B_1(c, v) \frac{\partial}{\partial c} \pi^S(c, v) + B_2(c, v) \pi^S(c, v) = 0$$

Second Order Condition

The second order condition is implied by our assumption on the type-elasticities:

$$\frac{\partial}{\partial \theta} \log (u(1, m, \theta) - u(0, m, \theta)) > \frac{\partial}{\partial \theta} \log \left(\frac{\partial}{\partial m} u(x, m, \theta) \right) = \text{constant}$$

Log Example: $u(x, m; \theta) = \theta x + \log(m)$

$M=1$

Private types c, v are iid uniformly on $[2, 100]$

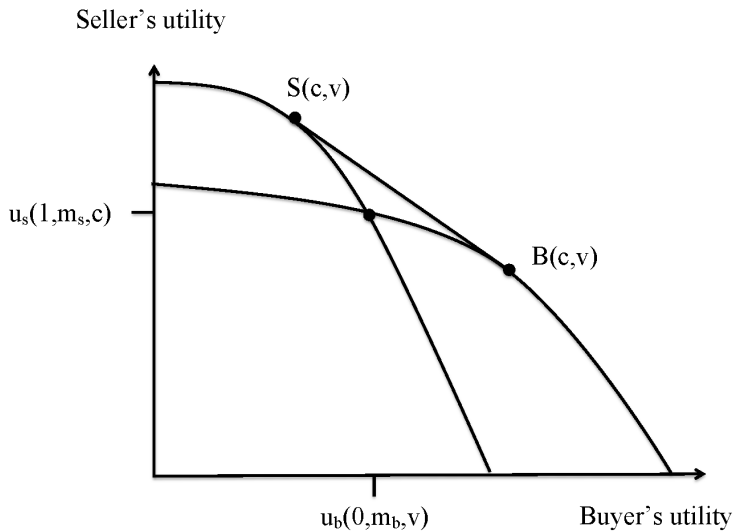
Choose m_b so that agents' utilities are equal for the mean profile of types.

$$51 + \log(1 - m_b) = \log(m_b).$$

Then, $m_b = \frac{e^{51}}{1+e^{51}}$ and $m_s = \frac{1}{1+e^{51}}$.

Efficient trade is possible!

Log Example: $u(x, m; \theta) = \theta x + \log(m)$



Log Example

When $u(x, m; \theta) = \theta x + \log(m)$ then:

- at point $S(c, v)$ the seller's money holdings are

$$m^S(c, v) = \frac{v}{c + v} M$$

- at point $B(c, v)$ the buyer's money holdings are

$$m^B(c, v) = \frac{c}{c + v} M$$

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Note: $M - m^S(c, v) = m^B(c, v)$. So money holdings for each player are the same in each state.

Log Example

The probability that the seller gets the item if the seller reports c and the buyer reports v is

$$\begin{aligned} \pi(c, v) = & \frac{1}{2} + \frac{1}{98} \int_{51}^c \frac{\log(100 + x) - \log(2 + x)}{x} dx \\ & + \frac{1}{98} \int_{51}^v \frac{-\log(100 + x) + \log(2 + x)}{x} dx. \end{aligned}$$

Log Example

First we verify that the mechanism is incentive compatible. Hence we can assume truthful reporting.

Then we verify that for any true types in the range, $[2, 100]$ the mechanism is individually rational.

Specifically, we need to show that for buyer and seller pairs with endowed wealths $m_b = \frac{e^{51}}{1+e^{51}}$ and $m_s = \frac{1}{1+e^{51}}$, and any type profile in $[2, 100]^2$, that both the buyer and the seller are better off under the mechanism than under no trade.

Log Example: IC

Under the assumption that the buyer truthfully reports her type, the seller optimally reports his true type, and vice versa. The mechanism achieves incentive compatibility by offsetting changes in the money allocation that result from false reports with changes in the probability of obtaining the item.

To illustrate this imagine a seller of type 51 reports her true type. Then her expected payoff is

$$(.5 + \textit{“expected change in probability due to buyer report”}) * 51 \\ + \textit{“expected value of consumption given truth”}$$

Log Example: IC

$$\begin{aligned} &= \left(.5 + \frac{1}{98} \int_2^{100} \frac{1}{98} \int_{51}^v \frac{-\log(100+x) + \log(2+x)}{x} dx dv \right) * 51 \\ &\quad + \int_2^{100} \log\left(\frac{v}{51+v}\right) \frac{1}{98} dv \\ &= 24.868001 \end{aligned}$$

Log Example: IC

If, in contrast, she reports 2 her expected payoff is

$$\begin{aligned} & (.5 + \text{"change in probability due to own misreport"} \\ & + \text{"expected change in probability due to buyer report"}) * 51 \\ & + \text{"expected value of consumption given lie"} \end{aligned}$$

Log Example: IC

$$\begin{aligned} &= \left(.5 + \frac{1}{98} \int_{51}^2 \frac{\log(100+x) - \log(2+x)}{x} dx \right. \\ &\quad \left. + \int_2^{100} \frac{1}{98} \int_{51}^v \frac{-\log(100+x) + \log(2+x)}{x} dx \frac{1}{98} dv \right) 51 \\ &\quad + \int_2^{100} \log\left(\frac{v}{2+v}\right) \frac{1}{98} dv \\ &= 22.010769. \end{aligned}$$

Log Example: IC

Why is misreporting costly?

If the buyer tells the truth she receives the item with probability 0.5052 and her expected utility from money holdings is -0.8985

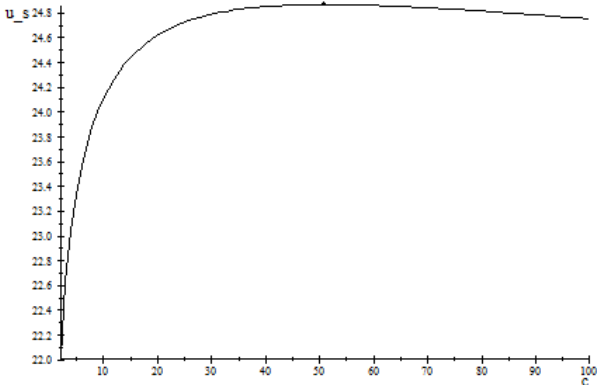
Recall: the seller's money holdings in either state are $\frac{v}{c+v}M$

If she deviates and reports type=2, she receives the item with probability 0.4330 and her expected utility from money holdings is -0.0722

Report	Money	Probability	Expected Payoff
2	↑	↓	↓

Log Example: IC

Of, course we need this to be true for any true type and any deviation. The following plot shows no deviation is profitable when the seller's true type is 51.



Log Example: IR

The expected utility of the type c seller under the mechanism is

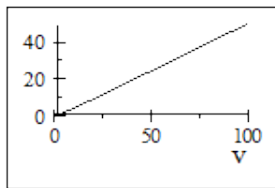
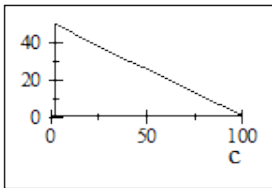
$$\frac{1}{98} \int_2^{100} \pi(c, v) c + \log\left(\frac{v}{c+v}\right) dv.$$

Similar expression for the buyer.

Log Example: IR

The no-trade payoffs for the seller and buyer are $c + \log\left(\frac{1}{1+e^{51}}\right)$, and $\log\left(\frac{e^{51}}{1+e^{51}}\right)$, respectively.

The following plots show that both functions are always non-negative.



Net Utility Gain for the Seller Net Utility Gain for the Buyer

Note that the expected net benefit to the seller at $c = 100$ and the buyer at $v = 2$ is $0.7938 > 0$.

Impossibility of Ex Post Implementation

When m^S, m^B are interior and efficiency requires randomization, then generically no mechanism is ex-post incentive compatible, individually rational, and implements efficient trade.

Conclusion

Eliciting money holdings

Public good provision

Conclusion

We show that efficient trade is possible in a natural class of environments without quasilinear utilities.

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We show that efficient trade is possible in a natural class of environments without quasilinear utilities.

New techniques to study mechanism design beyond the quasilinear environment.

Thank You